Flexibility of triangle-free planar graphs

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List coloring and choosability

List coloring

A list assignment for a graph G is a function that to each vertex $v \in V(G)$ assigns a set $L(v)$ of colors, and an L-coloring is a proper coloring ϕ such that $\phi(v) \in L(v)$ for all $v \in V(G)$.

Choosability

The choosability of graph G is the minimum integer k such that G has an L-coloring for every assignment L of lists of size at least k.

Flexibility

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Request

A request for a graph G with a list assignment L is a function r with $\mathrm{dom}(r) \subseteq V(G)$ such that $r(v) \in L(v)$ for all $v \in \mathrm{dom}(r)$.

ε-satisfiable request

For $\varepsilon > 0$, a request r is ε-satisfiable if there exists an L-coloring ϕ of G such that $\phi(v) = r(v)$ for at least $\varepsilon|\mathrm{dom}(r)|$ vertices $v \in \mathrm{dom}(r)$.

ε-flexibility

We say that G with the list assignment L is ε-flexible if every request is ε-satisfiable.

Weighted flexibility

Weighted request

Let L be a list assignment for a graph G. A weighted request is a function w that to each pair (v, c) with $c \in L(v)$ assigns a nonnegative real number.

ε-satisfiable request

Let $w(G, L) = \sum_{v \in V(G), c \in L(v)} w(v, c)$. For $\varepsilon > 0$, we say that w is ε-satisfiable if there exists an L-coloring ϕ of G such that

$$\sum_{v \in V(G)} w(v, \phi(v)) \geq \varepsilon w(G, L).$$

Weighted ε-flexibility

We say that G with the list assignment L is weighted ε-flexible if every weighted request is ε-satisfiable.

Previous results

Theorem (Dvořák, Norin, Postle 16')

For every integer $d \geq 0$, there exists $\varepsilon > 0$ such that every d-degenerate graph with an assignment of lists of size at least $d+2$ is weighted ε-flexible.

Theorem (Dvořák, Norin, Postle 16')

For every integer $d \geq 0$, there exists $\varepsilon > 0$ as follows. If G is a graph of maximum average degree less than $d+1+2/(d+4)$ then G with an assignment of lists of size $d+2$ is weighted ε-flexible.

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Theorem (Dvořák, Norin, Postle 16')

There exists $\varepsilon > 0$ such that every planar graph G with an assignment of lists of size 6 is ~~weighted~~ ε-flexible.

Previous results

Corollary

There exists $\varepsilon > 0$ such that each planar graph

- of girth 4 with assignment of lists of size 5 is weighted ε-flexible.
- of girth 5 with assignment of lists of size 4 is weighted ε-flexible.
- of girth 10 with assignment of lists of size 3 is weighted ε-flexible.

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Euler's formula trivially implies that planar triangle-free graphs are 4-choosable and that planar graphs of girth 6 are 3-choosable.

Our results

Theorem (Dvořák, TM, Musílek, Pangrác 17+)

There exists $\varepsilon > 0$ such that each planar triangle-free graph with assignment of lists of size 4 is weighted ε-flexible.

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Theorem (Dvořák, TM, Musílek, Pangrác 17+)

There exists $\varepsilon > 0$ such that each planar graph of girth 6 with assignment of lists of size 3 is weighted ε-flexible.

Schema of the proof(s)

1. Lemma by Dvořák, Norin, Postle 16'
2. Reducible configuration
3. Discharging

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For all integers $g, k \geq 3$ and $b \geq 1$, there exists $\varepsilon > 0$ as follows. Let G be a graph of girth at least g. If for every $Z \subseteq V(G)$, the graph $G[Z]$ contains an induced $(g-3,k)$-reducible subgraph H with at most b vertices, then G with any assignment of lists of size at least k is weighted ε-flexible.

1. for every vertex $v \in V(H)$, H is L-colorable for every assignment L, such that $|L(v)| = 1$ and $|L(w)| \geq k - \deg(w) + \deg_H(w)$ for all $w \neq v$,
2. for every $(g-3)$-independent set S of size at most $k-2$, H is L-colorable for every assignment L such that $|L(v)| \geq k - \deg(w) + \deg_H(v) - 1$ for $v \in S$ and $|L(v)| \geq k - \deg(w) + \deg_H(v)$ otherwise.

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For all integers and $b \geq 1$, there exists $\varepsilon > 0$ as follows. Let G be a graph of girth 4. If for every $Z \subseteq V(G)$, the graph $G[Z]$ contains an induced $(1,4)$-reducible subgraph H with at most b vertices, then G with any assignment of lists of size at least 4 is weighted ε-flexible.

1. for every vertex $v \in V(H)$, H is L-colorable for every assignment L, such that $|L(v)| = 1$ and $|L(w)| \geq 4 - \deg(w) + \deg_H(w)$ for all $w \neq v$,
2. for every independent set S of size at most 2, H is L-colorable for every assignment L such that $|L(v)| \geq 4 - \deg(w) + \deg_H(v) - 1$ for $v \in S$ and $|L(v)| \geq 4 - \deg(w) + \deg_H(v)$ otherwise.

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- Vertices of degree at most 2,
- Two adjacent vertices of degree 3.

Schema of the proof(s)

1. Lemma by Dvořák, Norin, Postle 16'
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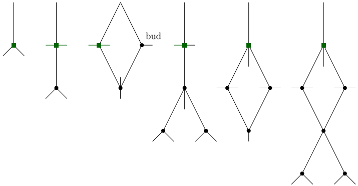
Let C be an outer face and each (≤ 5)-cycle bounds a face.
A vertex $v \notin V(C)$ of degree $d \geq 3$ st. it has $d-1$ (v, C)-good neighbors using distinct buds.



Schema of the proof(s)

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2. Reducible configuration
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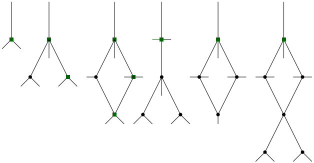
Let C be an outer face and each (≤ 5)-cycle bounds a face.
A vertex of G of degree 5 contained in a 4-face $vv_1v_2v_3$ such that $\deg(v_1) = \deg(v_3) = 3$, $\deg(v_2) = 4$, and $v, v_1, v_2, v_3 \notin V(C)$ st. it has a (v, C)-excellent neighbor x distinct from v_1 and v_3.



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Open questions

Does there exist $\varepsilon > 0$ such that every planar graph with anassignment of lists of size 5 is weighted ε-flexible? Or at least ε-flexible?

Does there exist $\varepsilon > 0$ such that every planar graph of girth at least 5 with an assignment of lists of size 3 is weighted ε-flexible? Or at least ε-flexible?

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Thank you for your attention!