Flexibility of Planar Graphs

Tomáš Masařík *Joint work with:* Zdeněk Dvořák, Jan Musílek, and Ondřej Pangrác

Definitions

A **weighted request** is a function w that to each pair (v, c) with $v \in V(G)$ and $c \in L(v)$ assigns a nonnegative real number.

Let $w(G, L) = \sum_{v\in V(G), c\in L(v)} w(v, c)$. For $\varepsilon > 0$, we say that w is **ε-satisfiable** if there exists an L-coloring φ of G such that

$$\sum_{v\in V(G)} w(v, \varphi(v)) \geq \varepsilon \cdot w(G, L).$$

We say that a graph G with the list assignment L is:

- **ε-flexible** if every request is ε-satisfiable,
- **weighted ε-flexible** if every weighted request is ε-satisfiable.

Previous Results

Dvořák, Norin, Postle: List coloring with requsts. JGT 19'

There exists $\varepsilon > 0$ such that every planar graph

- of girth at least 12 and with an assignment of lists of size 3 is **weighted** ε-flexible.
- of girth at least 5 and with an assignment of lists of size 4 is ε-flexible.
- with an assignment of lists of size 6 is ε-flexible.

Our Results

There exists $\varepsilon > 0$ such that every planar graph

- of girth at least 6 and with an assignment of lists of size 3 is **weighted** ε-flexible.
- without triangles and with an assignment of lists of size 4 is **weighted** ε-flexible.
- without 4-cycles and with an assignment of lists of size 5 is **weighted** ε-flexible.

Key Technique

For a function $f : V(G) \to \mathbb{Z}$ and a vertex $v \in V(H)$, let $\mathbf{f \downarrow v}$ denote the function such that $(f \downarrow v)(w) = f(w)$ for $w \neq v$ and $(f \downarrow v)(v) = 1$.

Suppose H is an induced subgraph of another graph G. For integers $k \geq 3$ and $d \geq 0$, we say that H is a **$(\mathbf{d}, \mathbf{k})$-reducible** induced subgraph of G if

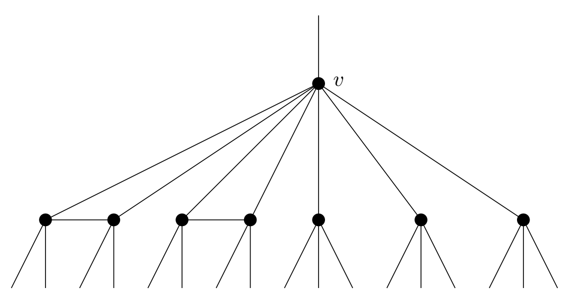
(FIX) for every $v \in V(H)$, H is L-colorable for every $((k + \deg_H - \deg_G) \downarrow v)$-assignment L, and

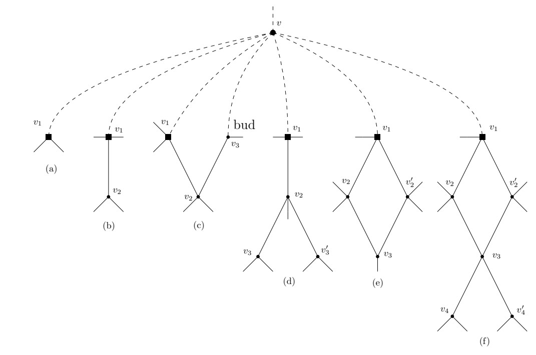
(FORB) for every d-independent set I in H of size at most $k - 2$, H is L-colorable for every $(k + \deg_H - \deg_G - 1_I)$-assignment L.

Lemma. For all integers $g, k \geq 3$ and $b \geq 1$, there exists $\varepsilon > 0$ as follows. Let G be a graph of girth at least g. If for every $Z \subseteq V(G)$, the graph $G[Z]$ contains an induced $(g - 3, k)$-reducible subgraph with at most b vertices, then G with any assignment of lists of size at least k is **weighted ε-flexible**.

Reducible Configurations

Without 4-cycle **Triangle-free**





Discharging. . .

Open Problems

Is there $\varepsilon > 0$ such that every planar graph

- with an assignment of lists of size <u>5</u> is (weighted) ε-flexible?
- of girth at least <u>5</u> and with an assignment of lists of size 3 is (weighted) ε-flexible?
- without 4-cycles and with an assignment of lists of size <u>4</u> is (weighted) ε-flexible?

- Zdeněk Dvořák, Tomáš Masařík, Jan Musílek, and Ondřej Pangrác: Flexibility of triangle-free planar graphs. arXiv:1902.02971 2019.
- Zdeněk Dvořák, Tomáš Masařík, Jan Musílek, and Ondřej Pangrác: Flexibility of planar graphs of girth at least six. arXiv:1902.04069 2019.
- Tomáš Masařík: Flexibility of planar graphs without 4-cycles. arXiv:1903.01460 2019 & Eurocomb 2019.

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