Flexibility of Planar Graphs

Tomáš Masařík *Joint work with:* Zdeněk Dvořák, Jan Musílek, and Ondřej Pangrác

Definitions

A **weighted request** is a function w that to each pair (v, c) with $v \in V(G)$ and $c \in L(v)$ assigns a nonnegative real number.

Let $w(G, L) = \sum_{v \in V(G), c \in L(v)} w(v, c)$. For $\varepsilon > 0$, we say that w is **ε-satisfiable** if there exists an L-coloring φ of G such that

$$\sum_{v \in V(G)} w(v, \varphi(v)) \geq \varepsilon \cdot w(G, L).$$

We say that a graph G with the list assignment L is:

- **ε-flexible** if every request is ε-satisfiable,
- **weighted ε-flexible** if every weighted request is ε-satisfiable.

Previous Results

Dvořák, Norin, Postle: List coloring with requsts. JGT 19'

There exists $\varepsilon > 0$ such that every planar graph

- of girth at least 5 and with an assignment of lists of size 4 is ε-flexible.
- of girth at least 12 and with an assignment of lists of size 5 is **weighted** ε-flexible.
- with an assignment of lists of size 6 is ε-flexible.

Our Results

There exists $\varepsilon > 0$ such that every planar graph

- of girth at least 6 and with an assignment of lists of size 3 is **weighted** ε-flexible.
- without triangles and with an assignment of lists of size 4 is **weighted** ε-flexible.
- without 4-cycles and with an assignment of lists of size 5 is **weighted** ε-flexible.

Key Technique

For a function $f : V(G) \to \mathbb{Z}$ and a vertex $v \in V(H)$, let $\mathbf{f} \downarrow \mathbf{v}$ denote the function such that $(f \downarrow v)(w) = f(w)$ for $w \neq v$ and $(f \downarrow v)(v) = 1$.

Suppose H is an induced subgraph of another graph G. For integers $k \geq 3$ and $d \geq 0$, we say that H is a **$(\mathbf{d}, \mathbf{k})$-reducible** induced subgraph of G if

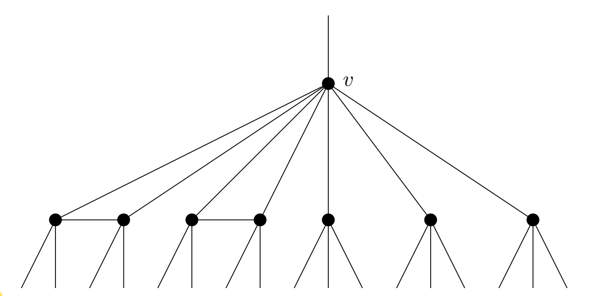
(FIX) for every $v \in V(H)$, H is L-colorable for every $((k + \deg_H - \deg_G) \downarrow v)$-assignment L, and

(FORB) for every d-independent set I in H of size at most $k - 2$, H is L-colorable for every $(k + \deg_H - \deg_G - 1_I)$-assignment L.

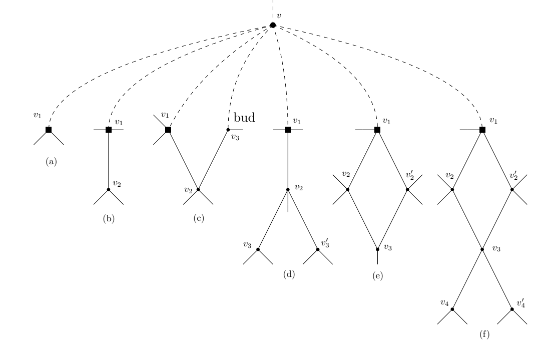
Lemma. For all integers $g, k \geq 3$ and $b \geq 1$, there exists $\varepsilon > 0$ as follows. Let G be a graph of girth at least g. If for every $Z \subseteq V(G)$, the graph $G[Z]$ contains an induced $(g - 3, k)$-reducible subgraph with at most b vertices, then G with any assignment of lists of size at least k is **weighted ε-flexible**.

Reducible Configurations

Without 4-cycle



Triangle-free



Discharging...

Open Problems

Is there $\varepsilon > 0$ such that every planar graph

- with an assignment of lists of size **5** is (weighted) ε-flexible?
- of girth at least **5** and with an assignment of lists of size 3 is (weighted) ε-flexible?
- without 4-cycles and with an assignment of lists of size **4** is (weighted) ε-flexible?

- Zdeněk Dvořák, Tomáš Masařík, Jan Musílek, and Ondřej Pangrác: Flexibility of triangle-free planar graphs. arXiv:1902.02971 2019.
- Zdeněk Dvořák, Tomáš Masařík, Jan Musílek, and Ondřej Pangrác: Flexibility of planar graphs of girth at least six. arXiv:1902.04069 2019.
- Tomáš Masařík: Flexibility of planar graphs without 4-cycles. arXiv:1903.01460 2019 & Eurocomb 2019.

UNIVERSITAS VARSOVIENSIS

Funding: One result of this research is a part of projects that have received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreements No 714704. The other two results are supported by project 17-04611S (Ramsey-like aspects of graph coloring) of Czech Science Foundation. This work was partially supported by student grants of Charles University GAUK 1277018 and SVV–2017–260452.