Flexibility of Planar Graphs Without 4-Cycles

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Bratislava, Slovakia

Related Definitions

List coloring

A list assignment for a graph G is a function that to each vertex $v \in V(G)$ assigns a set $L(v)$ of colors. An L-coloring ϕ is proper if $\phi(v) \in L(v)$ for all $v \in V(G)$ and $\phi(u) \neq \phi(v)$ for an edge u, v.

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The choosability of graph G is the minimum integer k such that G has an L-coloring for every assignment L of lists of size at least k.

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Precoloring Extension

The precoloring extension of graph G is a decision problem of extending a (pre)coloring of a subset of the vertices of a graph to a proper coloring of the whole graph.

Flexibility

Request

A request for a graph G with a list assignment L is a function r with $\mathrm{dom}(r) \subseteq V(G)$ such that $r(v) \in L(v)$ for all $v \in \mathrm{dom}(r)$.

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ε-satisfiable request

For $\varepsilon > 0$, a request r is ε-satisfiable if there exists an L-coloring ϕ of G such that $\phi(v) = r(v)$ for at least $\varepsilon|\mathrm{dom}(r)|$ vertices $v \in \mathrm{dom}(r)$.

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ε-flexibility

We say that G with the list assignment L is ε-flexible if every request is ε-satisfiable.

Weighted flexibility

Weighted request

Let L be a list assignment for a graph G. A weighted request is a function w that to each pair (v, c) with $c \in L(v)$ assigns a nonnegative real number.

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ε-satisfiable weighted request

Let $w(G, L) = \sum_{v \in V(G), c \in L(v)} w(v, c)$. For $\varepsilon > 0$, we say that w is ε-satisfiable if there exists an L-coloring ϕ of G such that

$$\sum_{v \in V(G)} w(v, \phi(v)) \geq \varepsilon w(G, L).$$

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Weighted ε-flexibility

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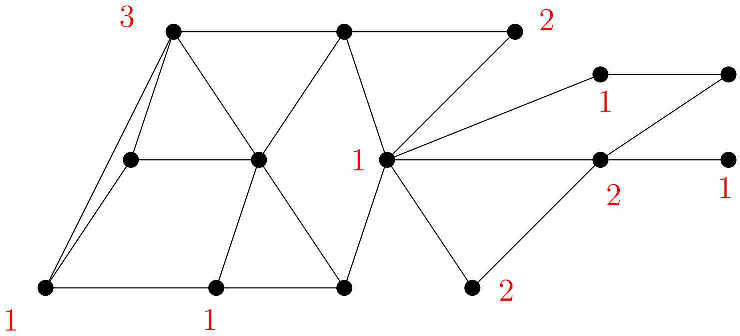

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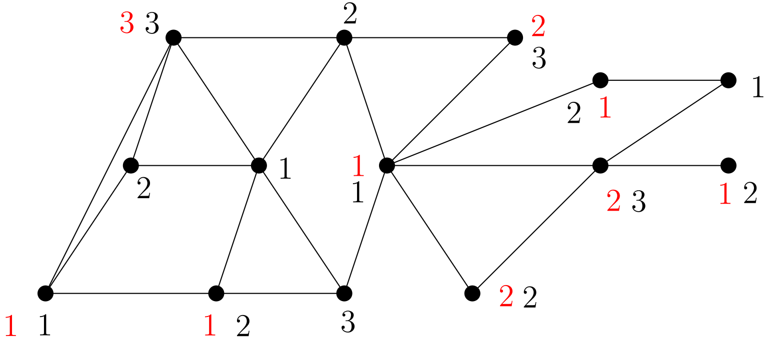

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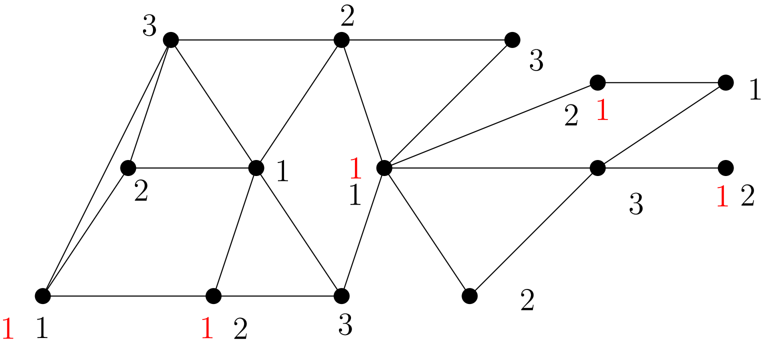

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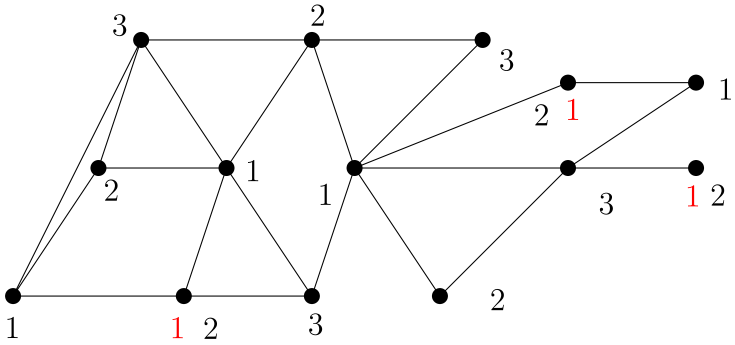

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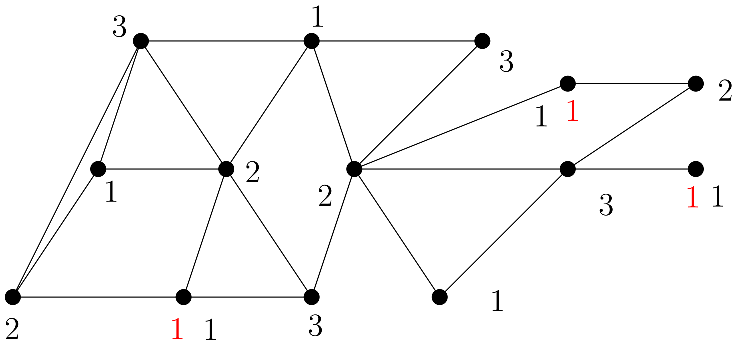

Our Result

Conjecture (Dvořák, Norin, Postle JGT 2019)

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There exists $\varepsilon > 0$ such that every planar graph with assignment of lists of size 5 is weighted ε-flexible.

Theorem (TM)

There exists $\varepsilon > 0$ such that every planar graph without C_4 with assignment of lists of size 5 is weighted ε-flexible.

Related Results

Planar		C_3	C_3, C_4	C_3, C_4, C_5	C_4
Ch. lb.	5 [Th]				
Ch.	5 [Th]				
W. Fl.	6 unw. [DNP]				

[Th] Thomassen 94
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Related Results

Planar		C_3	C_3, C_4	C_3, C_4, C_5	C_4
Ch. lb.	5 [Th]	4 [V]			
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Related Results

Planar		C_3	C_3, C_4	C_3, C_4, C_5	C_4
Ch. lb.	5 [Th]	4 [V]	3		
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Lemma (DNP 19, DMMP 19)

For all integers $b \geq 1$, there exists $\varepsilon > 0$ as follows. If for every $Z \subseteq V(G)$, the graph $G[Z]$ contains an induced $(0,k)$-reducible subgraph with at most b vertices, then G with any assignment of lists of size k is weighted ε-flexible.

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Vertex of degree $k-2$ is $(0,k)$-reducible.

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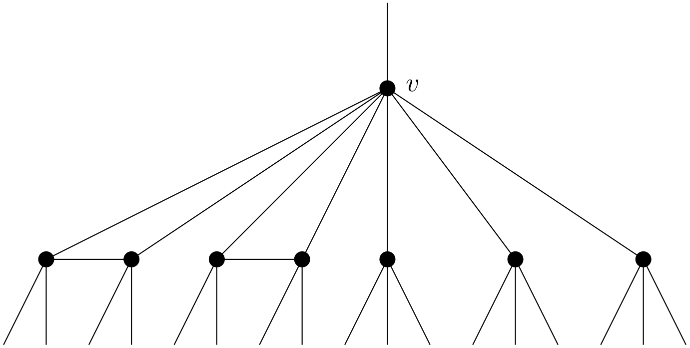
Vertex of degree 3 is $(0, 5)$-reducible.

Reducible Configuration



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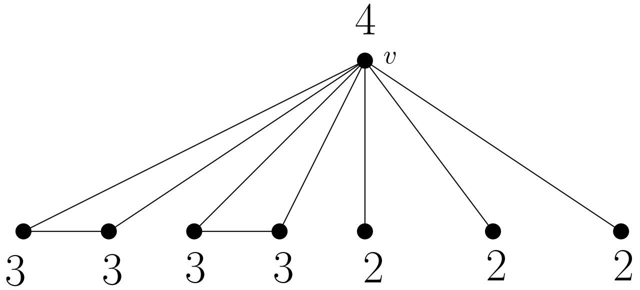

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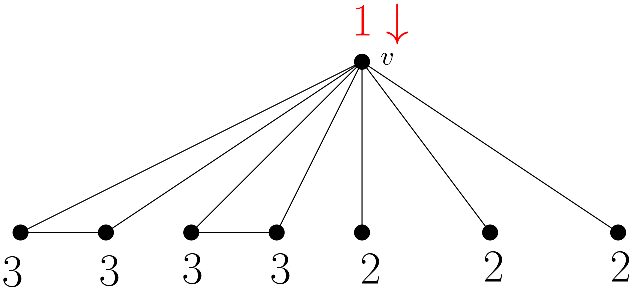

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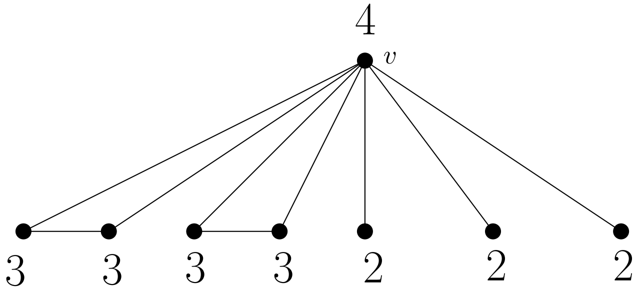

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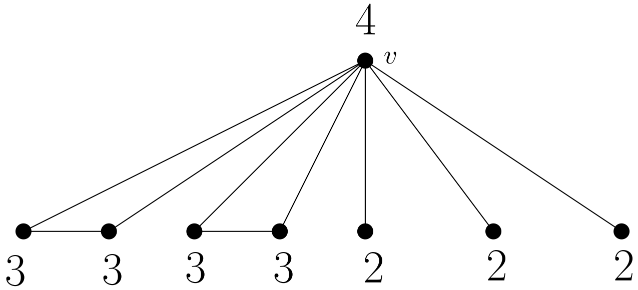

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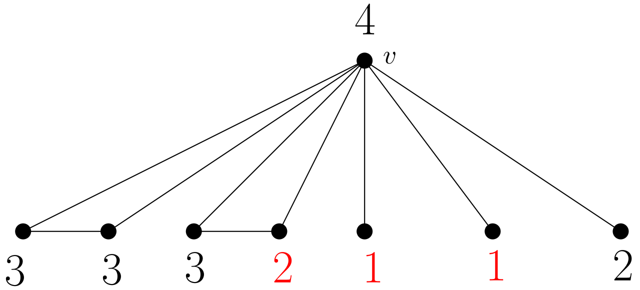

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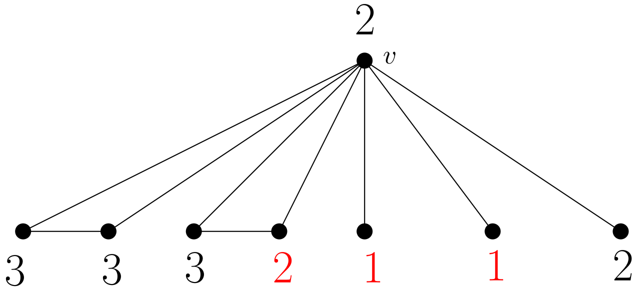

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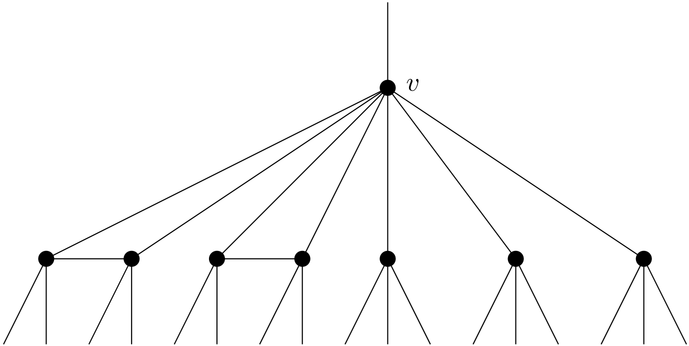

Discharging. . .

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are weighted ε-flexible?

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Lower bounds, algorithms...

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Thank you for your attention!