Parameterized complexity of fair deletion problems II.

Dušan Knop, Tomáš Masařík, Tomáš Toufar

Faculty of Mathematics and Physics,
Charles University,
Prague, Czech Republic.

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Toronto, Canada

CSI
IÚUK

Deletion problems

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Examples:

- Vertex Cover – $W \subseteq V$ such that $G \setminus W$ has no edges.
- Feedback Vertex set – $W \subseteq V$ such that $G \setminus W$ is a forest.
- Feedback Arc set – $F \subseteq E$ such that $G \setminus F$ is a DAG.
- Odd cycle transversal – $W \subseteq V$ such that $G \setminus W$ is a bipartite.
- Odd edge cycle transversal – $F \subseteq E$ such that $G \setminus F$ is a bipartite.

For monotone properties finding any such set is trivial.
Usual aim is to find the smallest such set.

Fair deletion problems

Usually aim is to find smallest set S such that $G \setminus S$ satisfies P.

In Fair deletion problems, we want to find set that is “locally” small.

- For a set F of edges, we want to minimize

$$\max_{v \in V} \deg_F(v).$$

- For a set W of vertices, we want to minimize

$$\max_{v \in V} |N(v) \cap W|.$$

Parameterized complexity

In parameterized complexity in addition to the input, we have a number called parameter.
Examples of parameters:

- size of the solution
- structural parameters (treewidth, clique width, vertex cover...)

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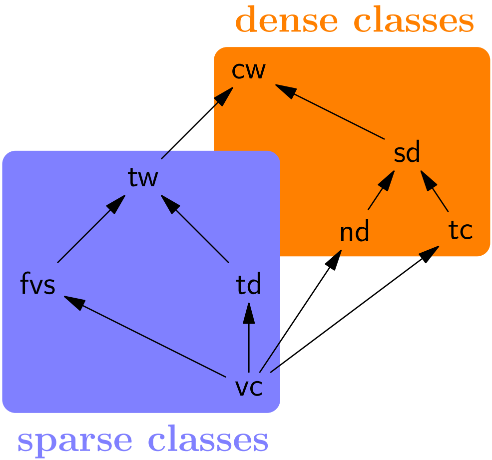
The running time is described as a function of both the size of the input and the parameter.

Complexity classes

- FPT – class of problems solvable in time $f(k)n^c$
- XP – class of problems solvable in time $n^{f(k)}$
- W[1]-hard –class of problems that unlikely admit an FPT alg.

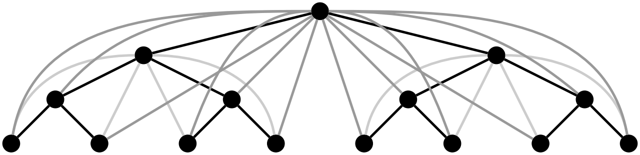
Overview of structural parameters



Tree depth

Definition (Tree depth)

- The closure $Clos(F)$ of a forest F is the graph obtained from F by making every vertex adjacent to all of its ancestors.
- The tree depth, denoted as $\mathrm{td}(G)$, of a graph G is one more than the minimum height of a rooted forest F such that $G \subseteq Clos(F)$.



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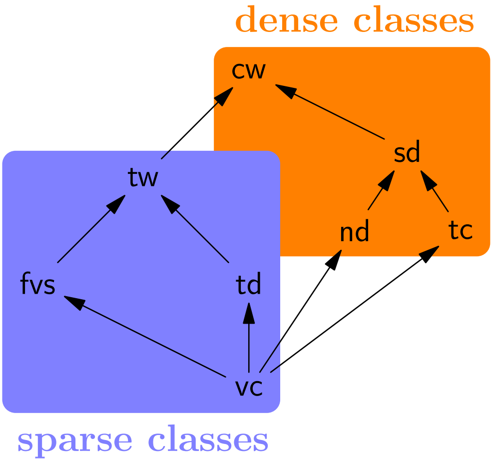
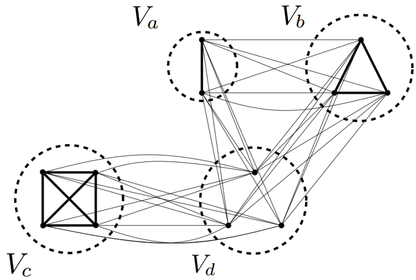


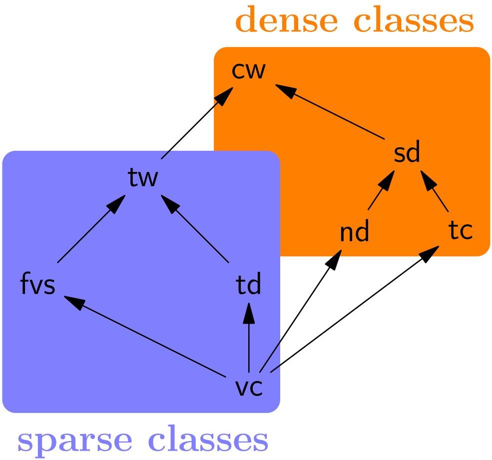
Neighborhood diversity

Definition (Neighborhood diversity)

The neighborhood diversity of a graph G is denoted by $\mathrm{nd}(G)$ and it is the minimum size of a partition of vertices into classes such that all vertices in the same class have the same neighborhood, i.e. $N(v) \setminus \{v'\} = N(v') \setminus \{v\}$, whenever v, v' are in the same class.



Overview of structural parameters



Graph properties

We study properties definable in graph logic (FO, MSO_1, MSO_2).

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Sometimes we want to put additional restriction on the deleted set itself (for example Connected vertex cover)
We use an MSO formula with one free set variable S such that $G \models \varphi(S)$ (in contrast to original $G \setminus S \models \psi$).

We call that generalized deletion problem or some authors use monadic second order evaluation.

Fair Vertex Cover

Problem formulation

Formulated as a Fair Vertex Deletion problem: Find a set of vertices W such that the rest of the graph has no edges.

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Our results

- FPT algorithm for the Fair vertex cover problem parameterized by modular width.
  (D. Knop, TM, T. Toufar 2017+)
- The Fair vertex cover problem is W[1]-hard parameterized by tree depth + feedback vertex set.
  (D.Knop, TM, T. Toufar 2017+)

Known results

- An XP algorithm for the generalized version of the Fair MSO_2 edge deletion problem parameterized by treewidth ($f(|\varphi|)n^{O(\mathrm{tw}(G))}$). Kolman, Lidický, and Sereni
- Can be easily adapted to the vertex version.

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- Can be easily adapted to the vertex version.
- MSO_2 does not admit XP algorithm on cliques unless standard complexity assumption fails by Lampis or Courcelle, Makowsky and Rotics.

Our results — Vertex deletion problem

Positive results

- FPT algorithm for generalized version of Fair MSO_1 vertex deletion problem parameterized by neighbourhood diversity. (TM, T. Toufar 2017)
- FPT algorithm for generalized version of Fair MSO_1 vertex deletion problem parameterized by twin cover. (D. Knop, TM, T.Toufar 2017+)

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Hardness results

- The Fair vertex cover problem is W[1]-hard parameterized by tree depth + feedback vertex set. (D.Knop, TM, T. Toufar 2017+)

Our results — Edge deletion problem

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Hardness results

- FO Fair deletion is W[1]-hard with respect to tree depth + feedback vertex set size.
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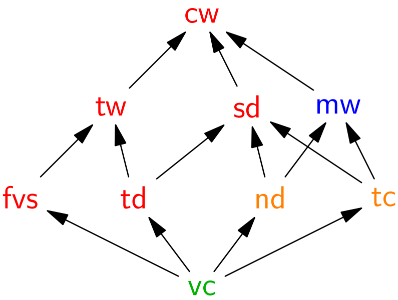
Overview of the results



Green means FPT algorithm for MSO_2 edge deletion problem.
Orange means FPT algorithm only for MSO_1 vertex deletion problem.
Red means hardness results for both edge and vertex deletion problem.
Blue means it is mostly unknown.

Open questions

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Thank you for your attention!