Maximum Weight Independent Set in Graphs with no Long Claws in Quasi-Polynomial Time

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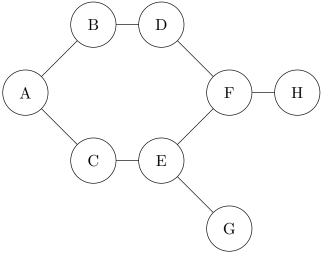
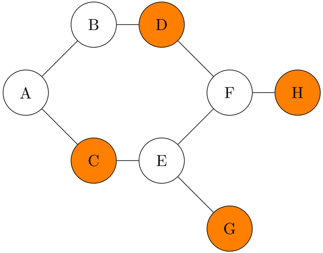
Max Weight Independent Set Problem

Definition (Max Weight Independent Set (MWIS))

Let G be a graph and let $\mathfrak{w} : V(G) \to \mathbb{N}$. The **MWIS** problem asks for a set $I \subseteq V(G)$ s.t. $G[I]$ is edgeless and $\mathfrak{w}(I)$ is as large as possible.





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Hereditary graphs: **DEF:** Graph classes closed under vertex-deletion operation.

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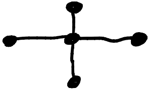
For **one** forbidden subgraph H ('82 Alekseev):

- Subdividing strategy proves NP-completeness when H is **not a forest** or **have two degree-three vertices** in one connected component.

- NP-complete when H does have **more than three leaves** in one connected component.



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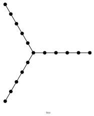
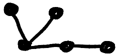
Let P_t be a path on t vertices.  Let $S_{t,t,t}$ be a $t-1$ times subdivided claw. 

Positive Results for MWIS

- **'08 Lozin, Milanič**
  ⇝ Polynomial on $S_{1,1,2}$-free graphs



- **'14 Lokshtanov, Vatshelle, and Villanger**
  ⇝ Polynomial on P_5-free graphs
- **'19 Grzesik, Klimošová, Pilipczuk, Pilipczuk**
  ⇝ Polynomial on P_6-free graphs
- **'20 Chudnovsky, Pilipczuk, Pilipczuk, Thomassé**
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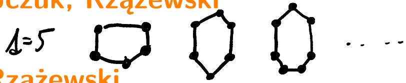
  ⇝ Quasi-polynomial on P_t-free graphs
- **'21 Gartland, Lokshtanov, Pilipczuk, Pilipczuk, Rzążewski**
  ⇝ Quasi-polynomial on $C_{\geq t}$-free graphs



- **'22 Abrishami, Chudnovsky, Dibek, and Rzążewski**
  ⇝ Polynomial on $S_{t,t,t}$-free graphs of bounded degree

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Theorem (MWIS in Quasipolynomial Time [GLMPPR '23])

For every H that is a forest whose every component has at most three leaves, there is an algorithm for the Maximum Weight Independent Set *problem in H-free graphs running in time $\mathbf{n}^{\mathcal{O}_{\mathbf{H}}(\log^{\mathbf{19}} \mathbf{n})}$.*

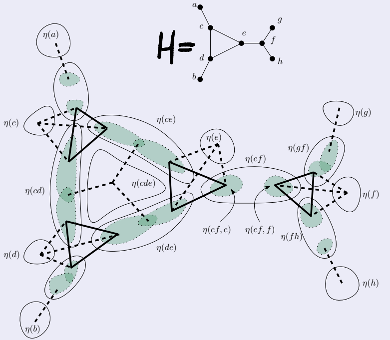
Proof—Structural Part

Structural part

- **Tool 1: Extended strip decomposition [Chudnovsky & Seymour '10]**

An *extended strip decomposition* of a graph G is a pair (H, η), where H is a simple graph and $\eta(\cdot) \subseteq V(G)$, such that.

1. $\{\eta(o) \mid o \in V(H) \cup E(H) \cup T(H)\}$ is a **partition** of $V(G)$.
2. For every $x \in V(H)$ and every distinct $y, z \in N_H(x)$, the set $\eta(xy, x)$ is **complete** to $\eta(xz, x)$.
3. Every $uv \in E(G)$ is **contained** in one of the sets $\eta(\cdot)$ or
   - $u \in \eta(xy, x), v \in \eta(xz, x)$ for some $x \in V(H)$ and $y, z \in N_H(x)$, or
   - $u \in \eta(xy, x), v \in \eta(x)$
   - $u \in \eta(xyz)$ and $v \in \eta(xy, x) \cap \eta(xy, y)$ for some $xyz \in T(H)$.



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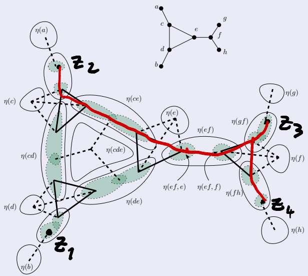
- **Tool 2: 3-in-a-tree theorem [Chudnovsky & Seymour '10]**
  Theorem: Let G be an n-vertex graph and $Z \subseteq V(G)$ with $|Z| \geq 2$. There is an algorithm that runs in time $\mathcal{O}(n^5)$ and returns one of the following:
  - an **induced subtree** of G containing **at least three** elements of Z, or
  - a rigid **extended strip decomposition** (H, η) **of** (G, Z).

$Z = \{z_1, z_2, z_3, z_4\}$



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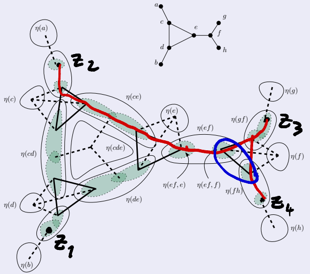

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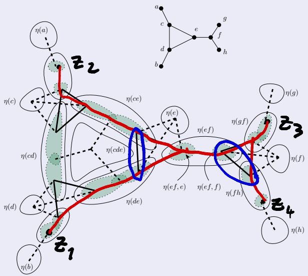

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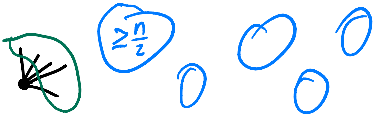

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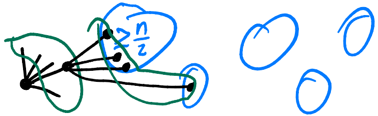

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In P_Δ-free graphs at most Δ-1 vertices.

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  1. an induced copy of $S_{t,t,t}$ in G;
  2. c_t**-dominated** 0.99τ**-balanced separator**;
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Proof Sketch—The Extended Strip Lemma

The extended strip lemma [GLMPPR '23]:

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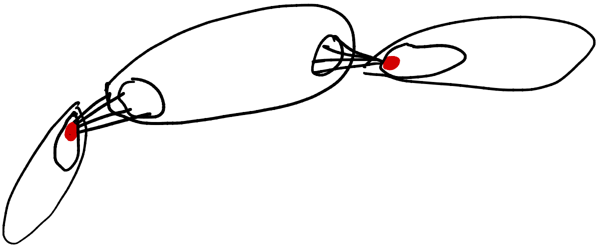
Observe: No particle has weight $\geq 0.01\tau$. If so ①.



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Otherwise: $\exists$ $O(1) \times O(1)$ wall st $\forall (A,B) \in \mathcal{T}_W$ B contains W and have weight $\geq 0.99\tau$

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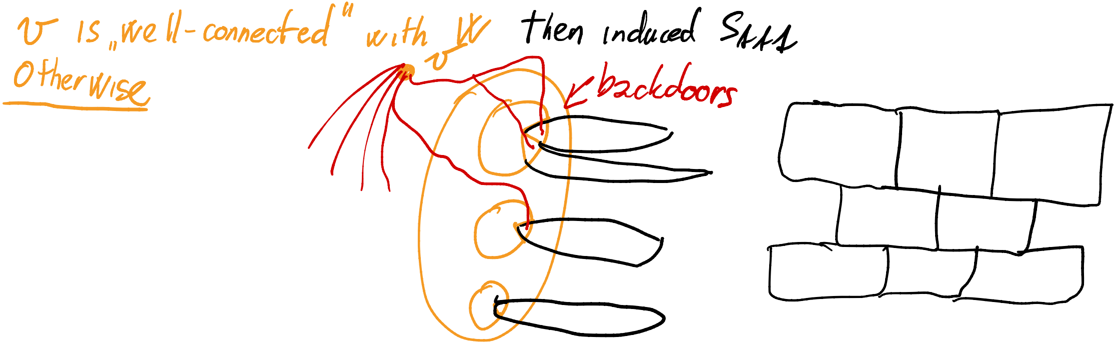

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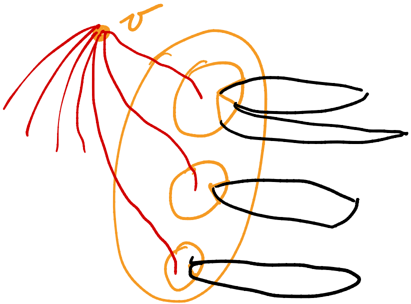

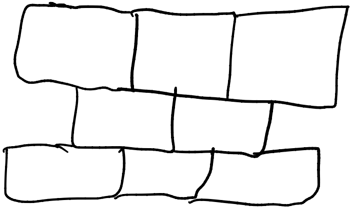

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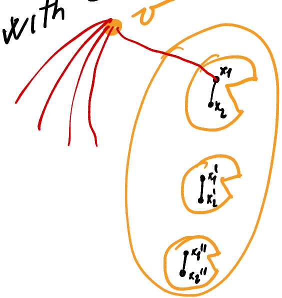
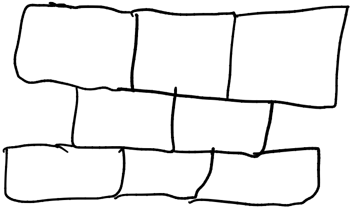
Use 3-in a tree on A with $Z = \{x_1\ x_2\ x_1'\ x_2'\ x_1''\ x_2''\}$

$w(A) \leq 0.01\tau$





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2. a **refined** extended strip decomposition of G.

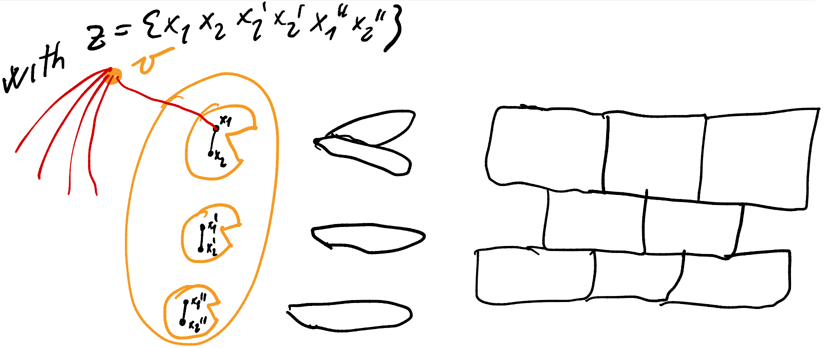
IF no backdoors

Use 3-in a tree on A with

$w(A) \leq 0.01\tau$

IF A has esd

then connect



Proof Sketch—The Extended Strip Lemma

The extended strip lemma [GLMPPR '23]:

For $t \in \mathbb{N}$ there exists $c_t \in \mathbb{N}$ and a **polynomial-time algorithm** that, given an n-vertex $S_{t,t,t}$-free graph G, a weight function $\mathfrak{w} : V(G) \to [0, +\infty)$, a real $\tau \geq \mathfrak{w}(G)$, a $v \in V(G)$, and a **refined extended strip decomposition** (H, η) of $G - v$, returns one of the following:

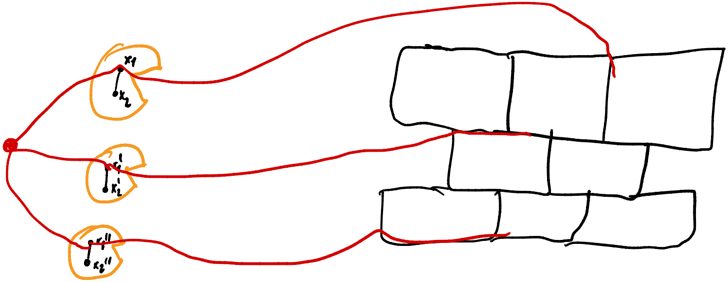
1. c_t**-dominated** 0.99τ**-balanced separator**;
2. a **refined** extended strip decomposition of G.

IF no backdoors

Use 3-in a tree on A with $Z = \{x_1 x_2 x_1' x_2' x_1'' x_2''\}$

$w(A) \leq 0.01\tau$

IF A has induced tree on Z. then demonstrate S_{ttt}



Proof—Algorithmic Part

k-dominated b-balanced separators

- **DEF:** Set $S \subseteq V(G)$ such that no component of $G - S$ has more than b vertices (or weight $\leq b$) and S is **dominated** by k vertices.
- Used to show quasipolynomial-time algorithm on P_t-free graphs **[Gartland&Lokshtanov '21]**
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s-boosted balanced separator

Simplified DEF: a set $N[S]$ dominated by a set S of at most s **vertices**, such that no component of $G - N[S]$ has more than $|V(G)|/16s^2$ vertices.

Packing lemma: Let G be an n-vertex $S_{t,t,t}$-free graph, $s \in \mathbb{N}$, and $\mathcal{F}$ a multi-set of subsets of $V(G)$ such that every set in $\mathcal{F}$ is an s-**boosted** balanced separator. Assume no vertex belongs to more **than** c **sets of** $\mathcal{F}$. Then, provided $|\mathcal{F}| \geq 80sct$, no component of G contains over $3n/4$ vertices.

$\rightsquigarrow$ **Quasipolynomial** branching

- Packing lemma is true assuming only k-dominated b-balanced separators in P_t-**free** graphs

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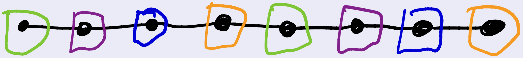

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- Packing lemma is true assuming only k-dominated b-balanced separators in P_t-**free** graphs

Boosting balanced separator

Boosting lemma: Let G be an n-vertex $S_{t,t,t}$-free graph, let $N[S]$ be a **balanced separator** for G dominated by a set S of at most c_t vertices, and let $\mathcal{F}$ be a multi-set of $|$**relevant**$(G,S)|/100c_t^3$-**balanced separators** for $(G, \text{relevant}(G,S))$. Assume no vertex belongs to more than c sets of $\mathcal{F}$. If $|\mathcal{F}| \geq 10ct$, either S is a c_t-**boosted balanced separator** or no component of G contains more than $3n/4$ vertices.

$\rightsquigarrow$ **Quasipolynomial** branching

Conclusions

Open questions

- Polynomial algorithm for MWIS on $S_{t,t,t}$**-free** graphs?

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Thank you for your attention!