Fair MSO Vertex Evaluation problems on graphs of bounded modular-width

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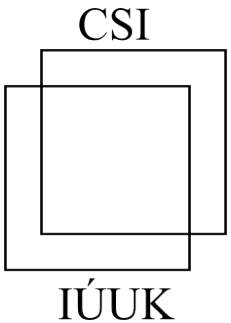
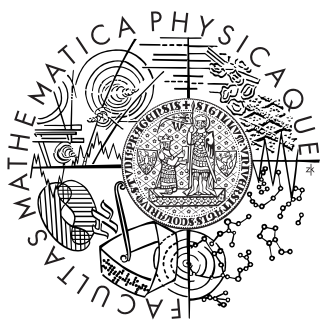
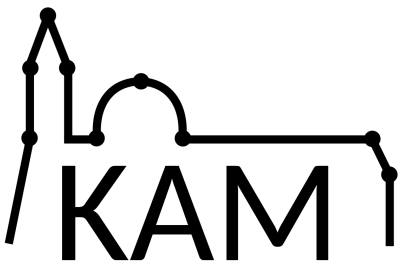
Open problems session ISAAC 2018,
Jiaoxi, Taiwan







Problem definition

Vertex Deletion problem [Yannakakis SIAM J comput. 81']

Given a graph $G = (V, E)$, find the smallest possible set of vertices W such that $G \setminus W$ satisfies the property π.

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Fair L Vertex Deletion	
Input:	An undirected graph $G = (V, E)$ and a positive integer k.
Fixed:	An L sentence φ.
Question:	Is there a set $W \subseteq V$ such that $G \setminus W \models \varphi$ with fair cost at most k?

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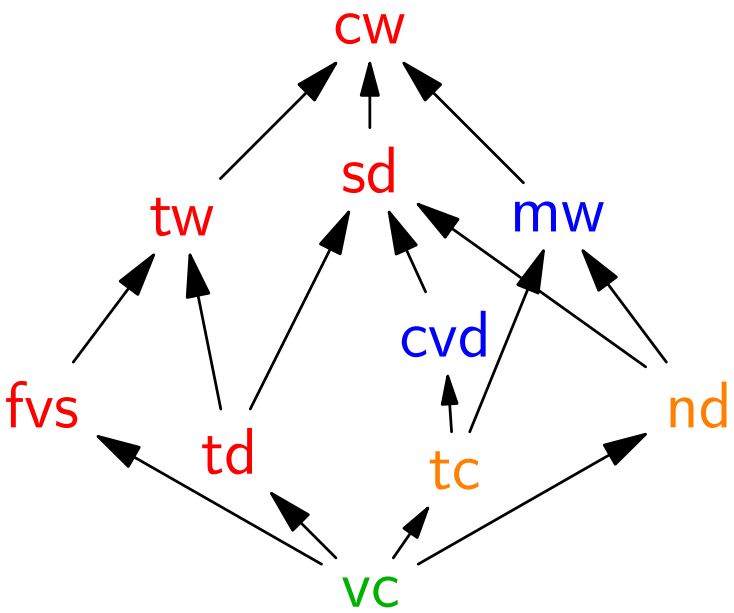
Fair L Vertex Evaluation

Input:	An undirected graph $G = (V, E)$ and a positive integer k.
Fixed:	An L formula $\varphi(X)$ with one free variable.
Question:	Is there a set $W \subseteq V$ such that $G \models \varphi(W)$ having fair cost at most k?

Known Results — overview



[1] Dušan Knop, Tomáš Masařík and Tomáš Toufar.
Parameterized Complexity of Fair Vertex Evaluation Problems.
arXiv:1803.06878, 2018+

[2] Tomáš Masařík and Tomáš Toufar.
Parameterized complexity of fair deletion problems.
TAMC 2017

[3] Dušan Knop, Martin Koutecký, Tomáš Masařík, and Tomáš Toufar.
Simplified algorithmic metatheorems beyond MSO: Treewidth and neighborhood diversity.
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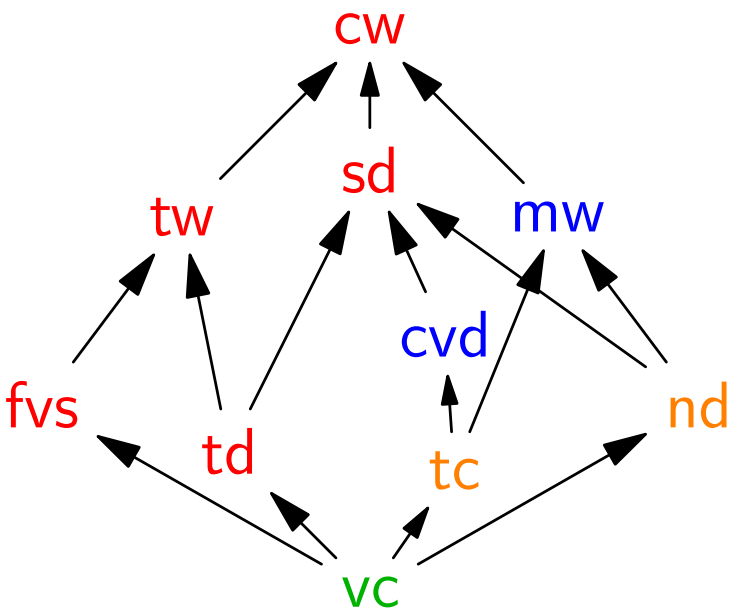
	vc	fvs + td	tc	nd	cvd	mw
Fair VC	$*$	[1]	$*$	$*$	?	[1]
FV L Del	MSO_2 $*$	$\mathsf{L}_\emptyset$ $*$	MSO_1 $*$	MSO_1 $*$	?	?
FV L Eval	MSO_2 [2]	$\mathsf{L}_\emptyset$ $*$	MSO_1 [1]	MSO_1 $*$	?	?
ℓ-FV L Eval	MSO_1 $*$	$\mathsf{L}_\emptyset$ $*$	MSO_1 [1]	MSO_1 [3]	MSO_1 $*$	MSO_1 $*$
FE L Del	MSO_2 [2]	FO [2]	?	?	FO [1]	?

Known Results — overview



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FV L Eval	MSO_2 [2]	$\mathsf{L}_\emptyset$ $*$	MSO_1 [1]	MSO_1 $*$	?	?
ℓ-FV L Eval	MSO_1 $*$	$\mathsf{L}_\emptyset$ $*$	MSO_1 [1]	MSO_1 [3]	MSO_1 $*$	MSO_1 $*$
FE L Del	MSO_2 [2]	FO [2]	?	?	FO [1]	?

Thank you for your attention!